4.4

Break It Down

Factoring Higher Order Polynomials

LEARNING GOAL

In this lesson, you will:

- Factor higher order polynomials using a variety of factoring methods.

Factoring in mathematics is similar to the breakdown of physical and chemical properties in chemistry.

For example, the chemical formula of water is H_2O. This formula means that for every 2 molecules of hydrogen (H) combined with one molecule of oxygen (O) creates water. Something that we take for granted such as water gives us insight into its individual parts or factors.

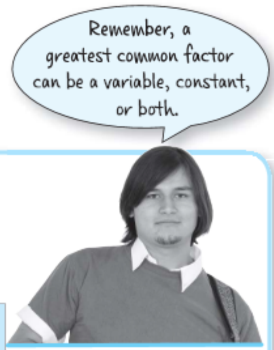
Although the general idea is the same between factoring in mathematics, and the breakdown of chemicals, there are some big differences. When factoring polynomials, the factored form does not change any of the characteristics of the polynomial; they are two equivalent expressions. The decomposition of chemicals, however, can sometimes cause an unwanted reaction.

If you have ever taken prescription medication, you might have read the warning labels giving specific directions on how to store the medication, including temperature and humidity. The reason for these directions: if the temperature is too hot or too cold, or if the air is too humid or too dry, the chemicals in the medication may begin to decompose; thus changing its properties.

What other reasons might people want to break down the chemical and physical components of things? How else can these breakdowns be beneficial to people?

PROBLEM 1 There's More Than One Way to Parse a Polynomial

In this lesson, you will explore different methods of factoring. To begin factoring any polynomial, always look for a greatest common factor (GCF). You can factor out the greatest common factor of the polynomial, and then factor what remains.



1. Ping and Shalisha each attempt to factor $3x^3 + 6x^2 - 15x$ by factoring out the greatest common factor.

Ping's Work

$3x^3 + 12x^2 - 36x$

$3x(x^2 + 4x - 12)$

Shalisha's Work

$3x^3 + 12x^2 - 36x$

$3(x^3 + 4x^2 - 12x)$

Analyze each student's work. Determine which student is correct and explain the inaccuracy in the other student's work.

2. If possible, completely factor the expression that Ping and Shalisha started.

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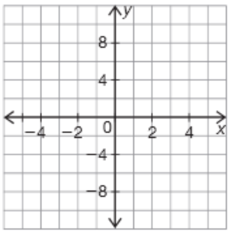
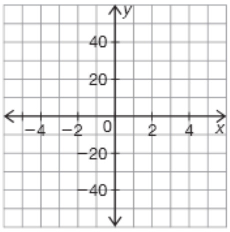
3. Factor each expression over the set of real numbers. Remember to look for a greatest common factor first. Then, use the factors to sketch the graph of each polynomial.

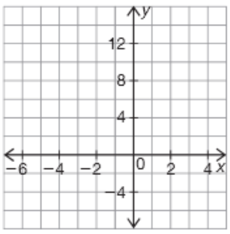
a. $3x^3 - 3x^2 - 6x$



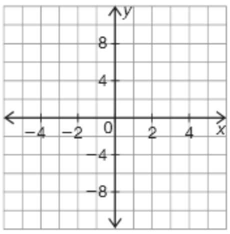
b. $x^3 - x^2 - 20x$



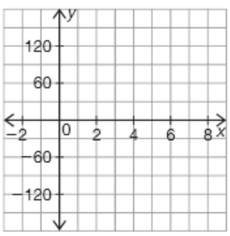
c. $2x^2 + 6x$



d. $3x^2 - 3x - 6$



e. $10x^2 - 50x - 60$



4. Analyze the factored form and the corresponding graphs in Question 3. What do the graphs in part (a) through part (c) have in common that the graphs of part (d) and part (e) do not? Explain your reasoning.

5. Write a statement about the graphs of all polynomials that have a monomial GCF that contains a variable.

PROBLEM 2 Continue Parsing

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Some polynomials in quadratic form may have common factors in some of the terms, but not all terms. In this case, it may be helpful to write the terms as a product of 2 terms. You can then substitute the common term with a variable, z, and factor as you would any polynomial in quadratic form. This method of factoring is called chunking.

You can use chunking to factor $9x^2 + 21x + 10$.

Notice that the first and second terms both contain the common factor, $3x$.

$9x^2 + 21x + 10 = (3x)^2 + 7(3x) + 10$	Rewrite terms as a product of common factors.
$= z^2 + 7z + 10$	Let $z = 3x$.
$= (z + 5)(z + 2)$	Factor the quadratic.
$= (3x + 5)(3x + 2)$	Substitute $3x$ for z.

The factors of $9x^2 + 21x + 10$ are $(3x + 5)(3x + 2)$.

1. Use chunking to factor $49x^2 + 35x + 6$.

2. Given $z^2 + 2z - 15 = (z - 3)(z + 5)$, write another polynomial in standard form that has a factored form of $(z - 3)(z + 5)$ with different values for z.

Using a similar method of factoring, you may notice, in polynomials with 4 terms, that although not all terms share a common factor, pairs of terms might share a common factor. In this situation, you can factor by grouping.

3. Colt factors the polynomial expression $x^3 + 3x^2 - x - 3$.

Colt

$x^3 + 3x^2 - x - 3$

$x^2(x + 3) - 1(x + 3)$

$(x + 3)(x^2 - 1)$

$(x + 3)(x + 1)(x - 1)$

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Explain the steps Colt took to factor the polynomial expression.

$x^3 + 3x^2 - x - 3$

$x^2(x + 3) - 1(x + 3)$ Step 1: ______________________________

____

$(x + 3)(x^2 - 1)$ Step 2: ______________________________

____

$(x + 3)(x + 1)(x - 1)$ Step 3: ______________________________

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4. Use factor by grouping to factor the polynomial expression $x^3 + 7x^2 - 4x - 28$.

5. Braxton and Kenny both factor the polynomial expression $x^3 + 2x^2 + 4x + 8$.

Braxton

$x^3 + 2x^2 + 4x + 8$

$x^2(x + 2) + 4(x + 2)$

$(x^2 + 4)(x + 2)$

Kenny

$x^3 + 2x^2 + 4x + 8$

$x^2(x + 2) + 4(x + 2)$

$(x^2 + 4)(x + 2)$

$(x + 2i)(x - 2i)(x + 2)$

Analyze the set of factors in each student's work. Describe the set of numbers over which each student factored.

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Recall that the Fundamental Theorem of Algebra states that any polynomial equation of degree n must have exactly n complex roots or solutions. Also, the Fundamental Theorem of Algebra states that every polynomial function of degree n must have exactly n complex zeros.

This implies that any polynomial function of degree n must have exactly n complex factors:

$f(x) = (x - r_1)(x - r_2) \ldots (x - r_n)$ where $r \in$ {*complex numbers*}.

Some 4th degree polynomials, written as a trinomial, look very similar to quadratics as they have the same form, $ax^4 + bx^2 + c$. When this is the case, the polynomial may be factored using the same methods as you would factor a quadratic. This is called factoring by using quadratic form.

Factor the quartic polynomial using quadratic form.

$x^4 - 29x^2 + 100$
- Determine whether you can factor the given trinomial into 2 factors.

$(x^2 - 4)\ (x^2 - 25)$
- Determine if you can continue to factor each binomial further.

$(x - 2)(x + 2)(x - 5)(x + 5)$

6. Factor each polynomial expression over the set of complex numbers.

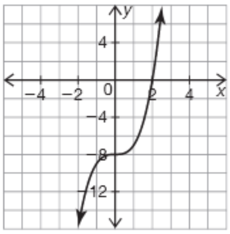
a. $x^4 - 4x^3 - x^2 + 4x$

b. $x^4 - 10x^2 + 9$

PROBLEM 3 Still Parsing

1. Factor each polynomial function over the set of real numbers.

a. $f(x) = x^3 - 8$



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b.

x	$f(x) = x^3 + 27$
−4	−37
−3	0
−2	19
−1	26
0	27
1	28

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You may have noticed that all the terms in the polynomials from Question 1 are perfect cubes. You can rewrite the expression $x^3 - 8$ as $(x)^3 - (2)^3$, and $x^3 + 27$ as $(x)^3 + (3)^3$. When you factor sums and differences of cubes, there is a special factoring formula you can use similar to the difference of squares in quadratics.

To determine the formula for the factored form, generalize the difference of cubes as $a^3 - b^3$.

To determine the factor formula for the difference of cubes, factor out $(a - b)$ by considering $(a^3 - b^3) \div (a - b)$.

$$\begin{array}{r|l} & a^2 + ab + b^2 \\ a - b & a^3 + 0 + 0 - b^3 \\ & \underline{a^3 - a^2b} \\ & a^2b + 0 \\ & \underline{a^2b - ab^2} \\ & ab^2 - b^3 \\ & \underline{ab^2 - b^3} \\ & 0 \end{array}$$

Therefore, the difference of cubes can be rewritten in factored form: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

2. Determine the formula for the sum of cubes by dividing $a^3 + b^3$ by $(a + b)$.

Remember that you can factor a binomial that has perfect square *a*- and *c*-values and no middle value using the difference of squares.

You can use the difference of squares when you have a binomial of the form $a^2 - b^2$.

The binomial $a^2 - b^2 = (a + b)(a - b)$.

3. Use the difference of squares to factor each binomial over the set of real numbers.

a. $x^2 - 64$

b. $x^4 - 16$

c. $x^8 - 1$

d. $x^4 - y^4$

Another special form of polynomial is the perfect square trinomials. Perfect square trinomials occur when the polynomial is a trinomial, and where the first and last terms are perfect squares and the middle term is equivalent to 2 times the product of the first and last term's square root.

Factoring a perfect square trinomial can occur in two forms:

$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 + 2ab + b^2 = (a + b)^2$$

4. Determine which of the polynomial expression(s) is a perfect square trinomial and write it as a sum or difference of squares. If it is not a perfect square trinomial, explain why.

   a. $x^4 + 14x^2 - 49$

   b. $16x^2 - 40x + 100$

   c. $64x^2 - 32x + 4$

   d. $9x^4 + 9x^2 + 1$

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Talk the Talk

You have used many different methods of factoring:

- Factoring Out the Greatest Common Factor
- Chunking
- Factoring by Grouping
- Factoring in Quadratic Form
- Sum and Difference of Cubes
- Difference of Squares
- Perfect Square Trinomials

1. Based on the form and characteristics, match each polynomial with the method of factoring you would use from the bulleted list given. Every method from the bulleted list should be used only once. Explain why you choose the factoring method for each polynomial. Finally, write the polynomial in factored form over the set of real numbers.

Polynomial	Method of Factoring	Reason	Factored Form
$3x^4 + 2x^2 - 8$			
$9x^2 - 16$			
$x^2 - 12x + 36$			
$x^3 - 64$			
$x^3 + 2x^2 + 7x + 14$			
$25x^2 - 30x - 7$			
$2x^4 + 10x^3 + 12x^2$			

Be prepared to share your solutions and methods.

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